Spring 2022 Math 208 M Midterm 1

NAME (First,Last) : . . . .

Student ID . . . .

UW email . . . .

- Please use the same name that appears in Canvas.
- IMPORTANT: Your exam will be scanned: DO NOT write within 1 cm of the edge. Make sure your writing is clear and dark enough.
- Write your NAME (first, last) on top of the second page of this exam.
- If you run out of space, continue your work on the back of the second page and indicate clearly on the problem page that you have done so.
- Unless stated otherwise, you **MUST** Justify your answers.
- Your work needs to be neat and legible.

Problem 1 Consider the following system :

$$\begin{cases} x + y - z = k \\ 2x + 3y + kz = 3k \\ x + ky + 3z = 2k \end{cases}$$

1. For which values of k does the system have no solutions?

2. For which values of k does the system have exactly one solution ?

3. For which values of k does the system have infinitely many solutions ?

NAME (First,Last) :

Problem 2 Let A be the 4×4 matrix with columns c_1, c_2, c_3, c_4. The matrix $B = \begin{pmatrix} | & | & | & | & a \\ | & | & | & | & b \\ c_1 & c_2 & c_3 & c_4 & c \\ | & | & | & | & d \end{pmatrix}$

(that is B is the 4×5 matrix that consists of A plus an additional fifth column $\begin{pmatrix} a \\ b \\ c \\ d \end{pmatrix}$)

reduces to $\begin{pmatrix} 1 & 1 & -1 & 2 & a \\ 0 & 1 & -3 & -2 & b-a \\ 0 & 0 & 0 & 2 & c-b+a \\ 0 & 0 & 0 & 0 & d-a+2b \end{pmatrix}$

1. Are c_1, c_2, c_3, and c_4 linearly independent ? Justify your answer.

2. Are c_1, c_2, c_4 linearly independent ? Justify your answer.

3. Give an example of a vector $b \in R^4$ that it is not in span (c_1, c_2, c_3, c_4) , or explain why this is not possible. If you give an example, you need to justify why your example works.

Problem 3 This problem has three unrelated parts.

1. Give an example of a linear system with two equations and three variables that has no solutions, or explain why this is not possible.

2. Give an example of a 3x3 matrix A that has linearly independent columns and can be reduced (by performing a sequence of elementary operations) to a matrix B that has linearly dependent columns, or explain why this is not possible.

3. Give an example of three non zero vectors u_1, u_2, u_3 in R^3 that are linearly dependent, but u_1 is not in span (u_2, u_3), or explain why this is not possible.